Kantrowitz - there is no limit

The ‘Kantrowitz Limit’ was discussed in Alpha, and it has been an important factor in the design of Hyperloop.

Hyperloop Alpha described how the air that is displaced by the the pod, needs to flow back through the annulus (the space between the pod and tube). Alpha then described the annulus flow as being similar to the flow through a nozzle or orifice, and that it could not exceed the speed of sound, the ‘Kantrowitz Limit’.

At low speed, less than 500 km/h, the air is incompressible and constant volume, like a liquid. So this back flow through the annulus is necessary, and it increases the airflow over the pod. But at high speeds, approaching the speed of sound, the air is very compressible, the volume changes, and the annulus flow is totally different.

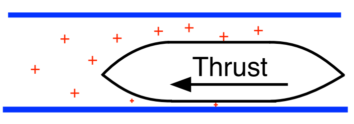
Alpha description of the airflow is correct at low speeds, but incorrect as the pod approaches the speed of sound. **It changes completely as the air become compressible**.

There are three different types of airflow at different speeds, Hyperloop is affected by them all.
Incompressible. Simple calculations such as Bernoulli can be used for cars and light aircraft
Compressible. Airliners fly at Mach 0.85, and the compression of the air must be considered.
Supersonic. Hyperloop may approach the speed of sound, depending on the gas in the tube.

In the examples below, it is assumed that the tube/pod area ratio is 2:1, so the pod area, and the annulus area, between the pod and tube, are equal. The speed of sound in air is 1,246 km/h and in steam is 1,539 km/h.

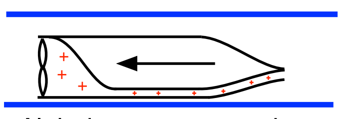
We consider external compression, not internal like Alpha.

Compression is required for Hyperloop to achieve any useful speeds in the confined space of the tube.



Pod with direct thrust, and external compression.

Alpha does this using the internal compressor, which forces the air through the relatively small duct. This consumes high power, due to the 30:1 compression ratio, and size and complexity of the compressor.


Alpha's compressor gives internal compresssion with a 30:1 ratio and high energy.

The more efficient solution is to compress the air outside the pod, in front of the pod, or in the annulus. The power is supplied by the thrust of the wheels or linear motors, which use the pod like a very streamlined piston.

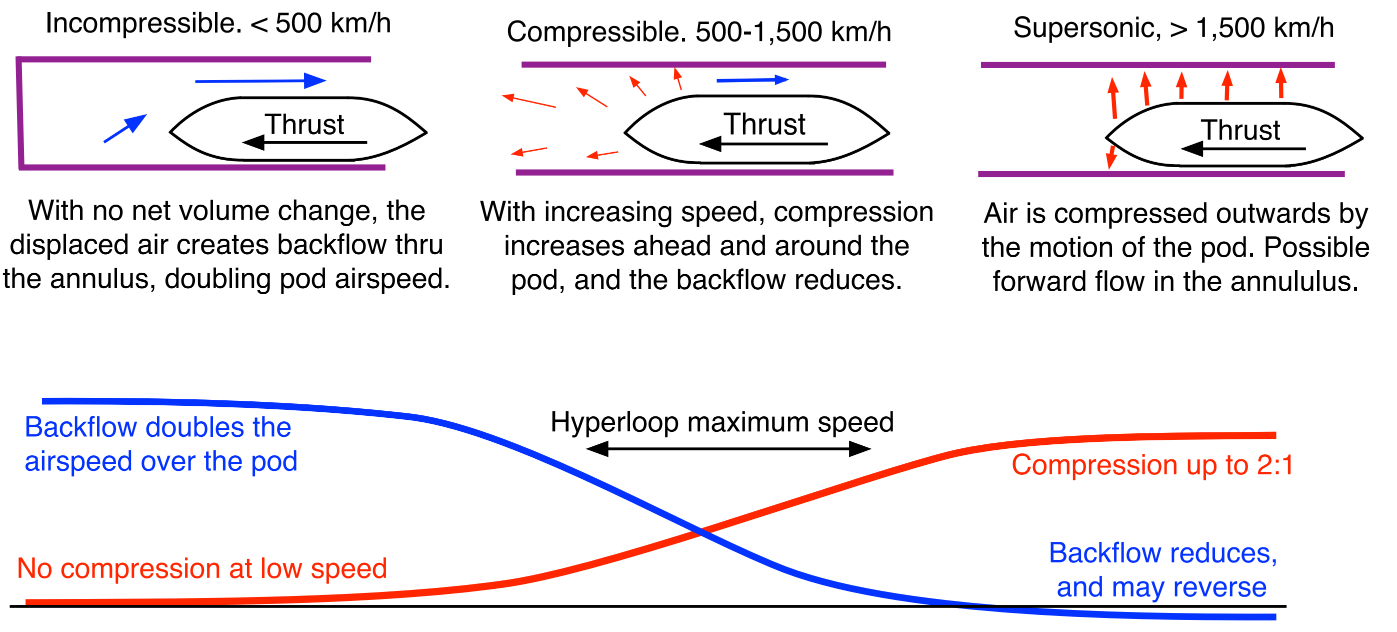
The simple streamlined pod with external compression is discussed here.

Hyperloop Alpha’s backflow theory only applies to low speed.

At low speed (< Mach 0.3), the air is not compressed, and has fixed volume. So the displaced air needs to flow backwards through the annulus, or forwards along the tube if possible. The annulus flow cannot exceed Mach 1, hence the ‘Kantrowitz Limit’

But at high speed (> Mach 0.5), the air is compressed, and the flow changes. The air is compressed outwards to let the pod pass, with a compression ratio less than 2:1.

Chart showing types of flow at different speeds



Incompressible flow
At low speeds, below about 500 km/h, the air is incompressible with constant volume. All the air displaced by the moving pod has to pass back through the annulus between the pod and tube. With a 2:1 area ratio, the air flows back at the same speed of the pod, so it would double the airspeed over the pod. The air in front of the pod is initially displaced forwards, then reverses its direction to flow backwards through the annulus.

The Alpha document only considered incompressible flow, and defined the ‘Kantrowitz Limit’ as a notional speed where the airspeed relative to the pod would become Mach 1. The pod speed is 620 km/h for air, and 770 km/h for steam.

Supersonic flow
Hyperloop will not be supersonic, but we need to to understand this type of flow to see what happens to Hyperloop at the high end of its speed range.

At high speeds, the air prefers to compress and reduce its volume, rather than travel any distance. So the flow is much simpler than the rather complex low speed flow. The air around the pod is simply pushed outwards, reducing its volume in order to let the pod pass.

The pod is like a projectile travelling in a confining tube. When a supersonic projectile or aircraft approaches, there is no sound or pressure wave ahead. So the air particles remain stationary, until they they are forced aside by the strong shock waves, which are instantaneous increases in pressure.

With a 2:1 area ratio, the air is compressed to double the pressure to allow the pod to pass. There would be some forward flow, due to the drag. Successive pods would move the air along the tube, increasing the pressure at the end, and creating a tailwind flow if the ends of the two tubes are vented together.

Compressible Flow
This is the area of interest for Hyperloop. As the pod speeds up, the flow will evolve from the incompressible back-flow model to the supersonic outward compression model.

A normal nozzle is restricted by the Kantrowitz Limit, where the flow becomes choked and cannot exceed the speed of sound. But we can force any amount of air through the nozzle, by increasing its pressure and density to increase the mass flow.

At the 300 to 800 km/h range, the drag in the annulus increases, and the air ahead of the pod will be compressed. This increases the density of the air, and reduces the volume and velocity through the annulus.

Over about 800 km/h, the compression ahead of the pod will transition to outward compression. The back-flow over the pod will reduce, then change to forward flow when supersonic. The compression ratio will increase up to a maximum of 2:1 at 1,500 km/h. So we could expect a compression ratio of 1.4 - 1.7 at 1,200 km/h. This is much lower than Alpha’s 30:1 compression ratio.

The Kantrowitz Limit theory is contradicted by supersonic guns.
If the Kantrowitz Limit theory was correct, then no projectile could approach or exceed Mach 1. But there are many supersonic guns, the fastest being the Light Gas Gun which accelerates projectiles to Mach 20.
So we need to consider compression when analysing Hyperloop’s airflow, rather than the incorrect Kantrowitz Limit which only considers incompressible, constant volume flow.

Conclusion
Hyperloop airflow studies have always assumed that the back flow in the annulus would increase, and the Kantrowitz Limit would eventually limit its speed, or require a considerably larger tube.
But the back flow reduces at high speeds, eventually changing to forward flow when approaching the speed of sound. So the drag and energy consumption of Hyperloop may be much lower than originally thought, and the compressor is not necessary. At 1,200 km/h, the flow over the pod may be Mach 0.9 for air, and much lower for steam.

So the **compressor on Alpha’s pod is not required**, and the tube can be smaller than has been proposed.

CFD Study
The flow around the pod is very complex, particularly in the mid speed range, where the flow is compressible, but not supersonic. A CFD analysis is required to accurately analyse the flow and drag at all speeds.